How large is a coulomb?

An elementary charge *e*, the charge held by one electron or one proton, has a value of 1.602 176 62(1) × 10^{-19} C (the parentheses indicate the error[1]). To understand how small this really is, consider that one coulomb must therefore have 6.241 509 34(14) × 10^{18} fundamental charges[2]. To see how much this is, consider a common rechargeable AAA battery (shown in Figure 1) that is capable of providing 850 mAh; that is, 0.85 A for one hour.

Figure 1. Rechargable AAA batteries from Duracell's web site.

Converting mAh to coulombs requires us to calculate

$$850\text{ mAh} = 850 \cdot 10^{-3}\frac{\text{C}}{\text{s}} \cdot \text{h} \cdot 3600\frac{\text{s}}{\text{h}} = 3060\text{ C}.$$

Thus, such a battery is capable of delivering on the order of 1.9 × 10^{22} fundamental electrical charges, or 19 *sextillion* fundamental electric charges. To consider how much charge can be delivered by one AAA battery, consider medium sand (what you probably had in your sand box) which has approximately 30,000 grains per cubic centimetre or 3 × 10^{10} grains of sand per cubic metre. 1.9 × 10^{22} grains of sand would occupy 6.4 × 10^{11} cubic metres or a cube of sand 8.6 km on each side. This much sand would create a conical pile[3] over 19 km across and 6.5 km high, the footprint of which is shown in Figure 2. Such a pile of sand would easily encompass all of Waterloo and Kitchener down to Highway 401.

Figure 2. The footprint of a pile of sand 19 km across (background maps are from maps.google.ca).

[1] The number in brackets indicates the uncertainty to one standard deviation in the last digit. Thus, the actual value is between 1.602 176 61 × 10^{-19} C and 1.602 176 63 × 10^{-19} C with 68 % certainty and between 1.602 176 60 × 10^{-19} C and 1.602 176 64 × 10^{-19} C (twice the width) with 95 % certainty.

[2] Again, the value lies between 6.241 509 20 × 10^{18} and 6.241 509 48 × 10^{18} fundamental charges with 68 % certainty.

[3] The *angle of repose* of a pile of dry sand is 34°.

Now, my favorite flashlight, the MAG-LITE XL200 LED, shown in Figure 3.

Figure 3. The MAG-LITE XL200 LED flashlight. Image from their web site.

This flashlight produces a maximum of 172 lumens and uses three AAA batteries. On high, this flashlight will drain its batteries after approximately two-and-a-half hours; thus, while running, it uses

$$\frac{3 \cdot 3060 \text{ C}}{2.5 \text{ h} \cdot \dfrac{3600 \text{ s}}{\text{h}}} = 1.02 \text{ A}$$

so almost exactly one coulomb per second. So much sand would create a pile over 1300 m across and 400 m high. Such a sand pile would fit comfortably within the north campus of the University of Waterloo and be more than nine times the height of the Dana Porter library, as shown in Figure 4.

Figure 4. The size of a pile of sand equivlaent to one approximately one coulomb.

Consequently, the number of grains of sand in such a pile would be approximately equal to the number of electrical charges in a coulomb.

On low, the flashlight drains its batteries after 218 h; thus, while running, it still produces a dim yet still useful light but using only

$$\frac{3 \cdot 3060 \text{ C}}{218 \text{ h} \cdot \dfrac{3600 \text{ s}}{\text{h}}} = 0.0117 \text{ A}$$

or 11.7 mA or 7.3×10^{16} charges per second. This would be equivalent to a pile of sand 300 m wide and 100 m high. Such a sand pile would have a footprint that fits comfortably over most of the engineering and physics buildings inside Ring Road and be over twice the height of the Dana Porter library.

Figure 5. The equivalent pile of sand to 11.7 mC.

Now, some of the leading low-power circuitry today uses on the order of a femtoampere (fA), in which case, this is a pile of sand approximately 13 mm across and 4.4 mm high being sent per second or 0.2 mL of sand per second.

Finding the size of a cone given a specific volume

To find he size of a cone, we use that the volume of a cone of radius r with height h is $\frac{1}{3}\pi r^2 h$ and the height of a pile of sand of radius r is $r\tan(34°)$ (as mentioned in an above footnote, the angle of repose of a pile of dry sand is 34°). Consequently, the formula that

$$V = \frac{1}{3}\pi r^3 \tan(34°)$$

so solving this for r yields

$$r = \sqrt[3]{\frac{3V}{\pi}\cot(34°)} \approx \frac{9}{8}\sqrt[3]{V} = 1.125\sqrt[3]{V}\ .$$

Also, $r\tan(34°) \approx 0.6745r \approx \frac{2}{3}r$.